$$① \ \frac{d}{dt}(\vec{A}\cdot\vec{A}) = \frac{d\vec{A}}{dt}\cdot\vec{A} + \vec{A}\cdot\frac{d\vec{A}}{dt}$$

$$\left[\frac{d}{dt}(\vec{A}\cdot\vec{B}) = \frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt}\right]$$

$$= \vec{A}\cdot\frac{d\vec{A}}{dt} + \vec{A}\cdot\frac{d\vec{A}}{dt} = 2\vec{A}\cdot\frac{d\vec{A}}{dt}$$

$$\left[\vec{A}\cdot\frac{d\vec{A}}{dt} = \frac{d\vec{A}}{dt}\cdot\vec{A}\right]$$

② $|\vec{A}| = \text{constant}$

Let $|\vec{A}| = c$ where c is a constant

Now $|\vec{A}|^2 = \vec{A}\cdot\vec{A}$

[এটা তোমরা জেনেও পারো যে
if $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$
হয় তাহলে $|\vec{A}| = \sqrt{x^2+y^2+z^2}$
$\vec{A}\cdot\vec{A} = x^2+y^2+z^2$]

$\Rightarrow \vec{A}\cdot\vec{A} = c^2$ [c = constant]

Differentiating with respect to t

$$\frac{d\vec{A}}{dt}\cdot\vec{A} + \vec{A}\cdot\frac{d\vec{A}}{dt} = 0$$

$\left[\because \frac{d}{dt}(\vec{A}\cdot\vec{B}) = \frac{d\vec{A}}{dt}\cdot\vec{B} + \vec{A}\cdot\frac{d\vec{B}}{dt}\right]$

$$\Rightarrow \frac{d\vec{A}}{dt}\cdot\vec{A} + \vec{A}\cdot\frac{d\vec{A}}{dt} = 0$$

$$\Rightarrow \vec{A}\cdot\frac{d\vec{A}}{dt} + \vec{A}\cdot\frac{d\vec{A}}{dt} = 0 \Rightarrow 2\vec{A}\cdot\frac{d\vec{A}}{dt} = 0$$

$$\Rightarrow \vec{A}\cdot\frac{d\vec{A}}{dt} = 0$$

[since scalar product commutative so
$\vec{A}\cdot\frac{d\vec{A}}{dt} = \frac{d\vec{A}}{dt}\cdot\vec{A}$]

$\Rightarrow |\vec{A}|\left|\frac{d\vec{A}}{dt}\right|\cos\theta = 0$ [θ angle between $\vec{A}$ and $\frac{d\vec{A}}{dt}$]

$\because \left|\frac{d\vec{A}}{dt}\right| \neq 0,\ |\vec{A}| \neq 0$

So $\cos\theta = 0 \Rightarrow \theta = 90^\circ$

$\vec{A}$ is perpendicular to $\frac{d\vec{A}}{dt}$